Debt-Dependent Effects of Fiscal Expansions

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Abstract

Economists often postulate that fiscal expansions are less stimulative when government debt is high than when it is low. Empirical evidence, however, is ambiguous. Using a nonlinear neoclassical growth model, we show that the difference in government spending effects between high- and low-debt environments depends on the wealth effect on labor supply and on whether the government uses taxes or spending to retire debt. Because of interrelated state variables, structural VAR estimations conditioning on debt alone can fail to isolate debt-dependent effects. Also, uncertainty on when the government will conduct fiscal consolidations generates wide confidence bands for spending multipliers, further complicating efforts to estimate debt-dependent government spending effects.

Keywords: debt-dependent government spending effect, state-dependent fiscal policy effects, fiscal multipliers, fiscal uncertainty
JEL codes: E62, H30, H60

1. Introduction

After the global financial crisis, many advanced economies entered an era of high government debt and slow growth. The average net government debt-to-GDP share increased from 44 percent in 2007 to 70 percent in 2014 for advanced economies (International Monetary Fund (2015)). Among G7 countries, excluding Canada and Germany, the average net government debt was 97 percent of GDP in 2014. Despite significant fiscal and monetary stimulus, the International Monetary Fund (2015) estimates that the output gap was −1.9 percent of potential GDP for advanced economies (−2.8 percent

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for the euro area) in 2014. Weak economic performance normally calls for expansionary fiscal policy. A high-debt environment, however, can undermine fiscal policy as an effective tool to combat recession.

Can government debt (hereinafter "debt") accumulation affect fiscal policy effects? The conventional view is that a fiscal expansion is less stimulative when debt is high than when it is low: a high-debt level induces expectations of high future taxes, discouraging current consumption, investment, and output. Empirical evidence, however, is ambiguous. Several papers, including Kirchner et al. (2010), Ilzetzki et al. (2013), and Nickel and Tudyka (2014), use large samples for OECD or other countries and find that the output multiplier is small or even negative in highly-indebted economies.[1] Using a similar dataset of OECD countries, Corsetti et al. (2012) do not find significant differences in output multipliers under various debt or deficits levels. When measuring fiscal conditions by sovereign yield spreads instead of debt levels, Born et al. (2015) find that a government spending cut has a negative effect on output under fiscal stress but a positive effect under no stress.[2] Such inconsistent findings could be driven by small differences in the debt-dependent effects or difficulty in isolating the effects.

In this paper, we first explore whether debt-dependent government spending effects are theoretically robust. They are not, particularly in the short run. The output multiplier difference between high- and low-debt states can be nontrival or trivial, depending on the wealth effect on labor supply and on whether the government uses tax increases or spending reversals to retire debt. We then show that commonly adopted VAR estima-

[1] Although Ilzetzki et al. (2013) report significant differences in the cumulative output multiplier between high- and low-debt countries for a longer horizon, the significance vanishes under an alternative specification as shown in their online appendix (Figure A11).

[2] Afonso and Jalles (2014) find similar results as Born et al. (2015) for consumption multipliers.

tions conditioning on debt alone are unlikely to isolate debt-dependent effects. Next, we allow for uncertain fiscal adjustment rules and find that uncertain fiscal consolidations generate wide confidence bands for spending multipliers.

We adopt a neoclassical growth model, where government spending enters households' utility as a complement to private consumption. This captures the short-run expansionary effects of government spending on consumption and output, as found in empirical work (see Blanchard and Perotti (2002), Perotti (2005), and Galí et al. (2007)). Fiscal multipliers are often computed by solving a linearized equilibrium system and, therefore, do not depend on debt levels (see Galí et al. (2007), Zubairy (2014), Leeper et al. (2015)). Instead, we obtain a fully non-linear solution under rational expectations, which allows us to explore government spending effects at different debt levels.[3] We find that, on average, an economy in a high-debt state has a smaller output multiplier than one in a low-debt state under the baseline specification with Greenwood, Hercowtiz, and Huffman's preference (GHH, Greenwood et al. (1988)) and with income taxes stabilizing debt. In the short run, the policy-expectations channel prevails; expecting higher future taxes discourages current consumption. Thus, a government spending increase is less expansionary on output. In the longer run, higher tax *levels* and a larger magnitude of fiscal adjustments in a high-debt state make a spending increase more contractionary on output than in a low-debt state.

The nontrivial differences in government spending effects between high- and low-debt states in the short run, however, are not robust to alternative model specifications. Since households derive utility from leisure, government spending effects also depend on

[3]By obtaining a fully non-linear solution, we also address Parker's (2011) critique on studying linearized dynamics of fiscal policy effects.

the wealth effect on labor. The hybrid GHH preference with government spending in the utility function substantially weakens this effect. Under King, Plosser, and Rebelo's preference (KPR, King et al. (1988)), we find that the stronger wealth effect on labor from expecting higher taxes in a high-debt state induces households to work harder initially than in a low-debt state, reversing the short-run labor response pattern observed under the hybrid GHH preference. Next, we allow both income tax rates and government spending to stabilize debt. Expecting lower future government spending then induces a positive wealth effect, making consumption and output rise more in a high-debt state than in a low-debt state, shrinking the short-run differences between two debt states.

Aside from different preferences and adjustment instruments, we also study how the capital state can affect debt-dependent government spending effects. At each period, an economy off the steady state inherits a set of state variables that are jointly affected by the history of economic and policy shocks. Historically, capital and debt are negatively correlated, although moderately. Thus, conditioning on debt alone may not be sufficient to recover the debt-dependent government spending effects. To show this, we perform structural VAR estimations on simulated data, following the standard structural VAR methodology in estimating fiscal policy effects. The simulated data from the baseline specification exhibit nontrivial debt-dependent effects in both the short and longer runs. Our estimations show that separating samples based on debt-to-output ratios (thus conditioning on the debt state only) fails to recover the true debt-dependent government spending effects.

Lastly, we explore policy uncertainty. Governments are often forced to conduct fiscal consolidations when debt is very high. When and how they do it, however, is often uncertain. To capture such uncertainty, we introduce an exogenous distribution of debt thresholds and model adjustment magnitudes to debt as a regime-switching process. At each period, an effective threshold is drawn from its distribution. If current indebtedness exceeds the effective threshold, which is more likely to occur with high debt, the government undertakes consolidations. Such policy uncertainty generates wide confidence bands surrounding output multipliers, further complicating efforts to estimate debt-dependent multipliers.

Our analysis adds to the literature on state-dependent fiscal policy effects, a topic enjoying substantial interest since the global financial crisis. The zero lower bound on the nominal interest rate and the deep recession have inspired research in fiscal policy effects conditional on different types of states, including monetary policy (e.g., Christiano et al. (2011) and Erceg and Lindé (2014)) and business cycles (e.g., Corsetti et al. (2010), Auerbach and Gorodnichenko (2012), Blanchard and Leigh (2013), and Owyang et al. (2013)). Our analysis is also related to papers that study how future fiscal adjustments can affect current fiscal policy effects through the intertemporal balancing of the government budget constraint, such as Dotsey (1994) and Dotsey and Mao (1997), but they do not focus on debt levels. Davig and Foerster (2015) study the impact of fiscal uncertainty with a known resolution date but a probabilistic adjustment magnitude, like the US "fiscal cliff," and find such uncertainty can generate volatility in the economy. Also, Bi et al. (2013) explore whether expansionary fiscal consolidations are likely for a high-debt economy. They adopt a New Keynesian model but omit capital, important for debt-dependent government spending effects as shown in our analysis.

2. Model Setup

We adopt a neoclassical growth model. The utility function allows for variation in the wealth effect on labor supply, similar to Monacelli and Perotti (2008) and Jaimovich and Rebelo (2009), and therefore can accommodate both the GHH (Greenwood et al. (1988)) and KPR preferences (King et al. (1988)). Our baseline specification features the GHH preference, motivated by Schmitt-Grohé and Uribe's (2012) estimates, and allows the income tax rate to stabilize debt. Later, we explore the KPR preference, government spending reversals as an adjustment instrument, and uncertain fiscal adjustments.

2.1. The Baseline Specification

Households derive utility from effective consumption ($\tilde{c}_t$) and disutility from labor (l_t). Effective consumption is a constant-elasticity-of-substitution (CES) index of private consumption (c_t) and government spending (g_t):

$$\tilde{c}_t = \left[\omega \left(c_t\right)^{\frac{\nu-1}{\nu}} + (1-\omega)\left(g_t\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{1}$$

as in Bouakez and Rebei (2007), where ω is the weight of private consumption in effective consumption, and $\nu > 0$ is the elasticity of substitution between private consumption and government spending. When $\nu \to 0$ ($\nu \to \infty$), government spending and private consumption are perfect complements (substitutes). We restrict c_t and g_t to be complements to capture the short-run stimulative effects of government spending on private consumption, as observed empirically.

A representative household chooses private consumption (c_t), labor (l_t), investment (i_t), and capital (k_t) to maximize expected utility

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{(\tilde{c}_t - \varphi l_t^{\theta} X_t)^{1-\sigma}}{1-\sigma}, \tag{2}$$

subject to the budget constraint

$$c_t + i_t + q_t b_t = (1 - \tau_t)\left(w_t l_t + r_t^k k_{t-1}\right) + b_{t-1} + z_t, \tag{3}$$

where b_t is a one-period government bond with a price q_t paying one unit of goods at $t+1$, τ_t is the income tax rate, w_t is the real wage rate, r_t^k is the rental rate for capital, and z_t is government transfers. To accommodate both the GHH and KPR preference specifications, we introduce an index variable X_t, which evolves according to

$$X_t = \tilde{c}_t^{\psi} X_{t-1}^{1-\psi}. \tag{4}$$

The parameter ψ determines the magnitude of the wealth effect on labor supply. If the government spending does not enter utility ($\omega = 1$), $\psi = 1$ implies the KPR preference, and $\psi = 0$ implies the GHH preference. By setting $\psi = 0$ and $\omega < 1$, we use a hybrid GHH preference for the baseline specification, which has government spending in the utility function and a small wealth effect on labor supply. This is different from the standard GHH preference, as the latter eliminates the wealth effect on labor supply.[4] When solving the baseline specification with $\psi = 0$, X_t is normalized to 1, following standard practice.

The law of motion for capital is

$$k_t = (1-\delta)k_{t-1} + i_t - \frac{\kappa}{2}\left(\frac{i_t}{k_{t-1}} - \delta\right)^2 k_{t-1}, \tag{5}$$

where $\frac{\kappa}{2}\left(\frac{i_t}{k_{t-1}} - \delta\right)^2 k_{t-1}$ is the capital adjustment cost.

Firms are perfectly competitive and produce with Cobb-Douglas technology:

$$y_t = a_t k_{t-1}^{\alpha} l_t^{1-\alpha}. \tag{6}$$

[4] Equations (A.1) to (A.4) in Appendix A show that the labor supply equilibrium condition is $\varphi\theta l_t^{\theta-1} = \omega c_t^{\frac{-1}{v}} \tilde{c}_t^{\frac{1}{v}} (1-\tau_t)\, w_t$. When $\omega \to 1$, $\tilde{c}_t \to c_t$, $\varphi\theta l_t^{\theta-1} = (1-\tau_t)\, w_t$, and the wealth effect on labor supply approaches zero.

Total factor productivity (TFP), a_t, follows an AR(1) process:

$$a_t = (1-\rho_a)a + \rho_a a_{t-1} + \varepsilon_t^a, \qquad \varepsilon_t^a \sim N(0,\sigma_a^2). \tag{7}$$

The government collects taxes and issues debt to pay for its purchases, transfers, and debt service. Its budget constraint is

$$tax_t + q_t b_t = g_t + b_{t-1} + z_t, \tag{8}$$

where $tax_t \equiv \tau_t \left(w_t l_t + r_t^k k_{t-1}\right) = \tau_t y_t$.

Periodic instability in transfers is common in many advanced economies and is likely to become more widespread as populations age. In the post-war U.S., rising transfers to households have become one of the main drivers of the debt growth (see Figure 1). We follow Bi et al. (2013) and assume an exogenous process for transfers with a regime-switching process:

$$z(rs_t^z) = \begin{cases} (1-\rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z, & \text{if } rs_t^z = 1, \qquad \rho_z < 1, \\ \mu z_{t-1} + \varepsilon_t^z, & \text{if } rs_t^z = 2, \qquad \mu > 1, \end{cases} \tag{9}$$

where $\varepsilon_t^z \sim N(0,\sigma_z^2)$ and the regime index rs_t^z evolves according to the transition matrix

$$\begin{pmatrix} p_1^z & 1-p_1^z \\ 1-p_2^z & p_2^z \end{pmatrix}.$$

This regime-switching process can generate rising transfers for prolonged periods that steadily raise debt.

Government spending follows the exogenous process

$$g_t = (1-\rho_g)\, g + \rho_g g_{t-1} + \varepsilon_t^g, \qquad \varepsilon_t^g \sim N(0,\sigma_g^2). \tag{10}$$

The income tax rate adjusts to stabilize debt:

$$\tau_t = \tau + \gamma_\tau (b_{t-1} - b). \tag{11}$$

2.2. Calibration and Solution Method

The model is calibrated at a quarterly frequency. The discount factor, β, is set to 0.99, and the depreciation rate, δ, is set to 0.025. Preference for consumption is logarithmic with $\sigma = 1$. The capital income share, α, is 0.36. To have a Frisch labor elasticity of 0.5, we set θ to 3. Following Gourio (2012), the capital adjustment parameter, κ, is 1.7. To calibrate effective consumption, $\tilde{c}_t$, we assume that the weight of private consumption, ω, is 0.9 for the U.S. economy. Since the elasticity of substitution between private consumption and government spending, ν, is not conventionally estimated, we back out $\nu = 0.42$ to have the model-implied impact (also peak) output multiplier to be about 0.8 when a government spending shock is given at the steady state.[5]

To calibrate the exogenous processes, we rely on reduced-form estimation using U.S. quarterly data from 1947Q1 to 2015Q2 from the National Income and Product Accounts (NIPA) Tables released by the Bureau of Economic Analysis.[6] Estimation of the regime-switching transfer rule, equation (9), yields $\rho_z = 0.960$ and $\mu = 1.006$. We set $p_1^z = p_2^z = 0.992$, which implies that the average length of each regime is 30 years. The fiscal data, except for debt, are from governments of all levels; the debt data are from the market value of privately held federal debt, published by the Federal Reserve Bank of Dallas.

[5] When estimating with the U.S. data (1960-2008) using a more elaborated neoclassical growth model, Leeper et al. (2010) report that the impact (also peak) output multiplier is about 0.6. Recent estimates using New Keynesian models that account for monetary and fiscal policy interaction yield much bigger short-run multipliers at 1.5 (Leeper et al. (2015)). Ramey's (2011) literature survey finds that the bulk of estimates of the output multiplier are between 0.8 and 1.5.

[6] Government spending is measured as the sum of current expenditure and gross government investment (NIPA Table 3.1, lines 20 and 39) minus net transfers and net interest payments (NIPA Table 3.1, lines 22 and 27). The income tax rate is measured by the ratio of current tax receipts (NIPA Table 3.1, line 2) plus contributions for government social insurance (NIPA Table 3.1, line 7) to GDP (NIPA Table 1.1.5, line 1). TFP is proxied by Solow residuals, estimated from GDP, total hours worked, and capital. Total hours are the product of the average working hour index and civilian employment. Average working hours are measured by the index of average weekly hours in nonfarm business (2005=100, seasonally adjusted, the Bureau of Labor Statistics (BLS), PRS85006023). Employment is measured by civilian employment for persons 16 years of age and older (seasonally adjusted, BLS, CE16OV). Capital is measured by the value of annual fixed assets and consumer durable goods (NIPA Fixed Assets Table 1.1, line 1), adjusted by the GDP deflator and interpolated to a quarterly series.

The baseline specification assumes that the income tax rate alone adjusts to debt. Based on the estimates by Leeper et al. (2010), we set $\gamma_\tau = 0.027$.[7] Calibrated to the sample averages in data, the steady-state government spending, transfers, and debt as a share of (annual) output are $\frac{g}{y} = 0.208$, $\frac{z}{y} = 0.086$, and $\frac{b}{4y} = 0.373$, which imply $\tau = 0.31$ from the government budget constraint, equation (8). Table 1 summarizes the calibration and some steady-state values.

To solve the model non-linearly, we use the monotone mapping method, as in Coleman (1990) and Davig (2004). Appendix A lists the equilibrium system for the baseline specification, and Appendix B describes the solution method.

3. Debt-Dependent Government Spending Effects

We analyze the effects of a government spending shock at various debt levels. The off-steady-state analysis raises an issue on how to pin down the relationships between the state of interest (debt) with the other state variable (capital) in the model economy. With multiple endogenous states, the economic structure and history of shocks make them interdependent. To account for these relationships, we use the model to simulate a joint distribution of capital and debt.

3.1. Simulating a State Distribution

The simulation starts from a uniform distribution of capital and debt at $t = -100$. The economy is subject to shocks to government spending, TFP, and transfers at each period from $t = -99$ to -1. We perform 5000 simulations. To remove outliers, only the simulations with a debt-output share falling within the 95-percent intervals of all the

[7] Leeper et al. (2010) specify fiscal adjustment rules in percentage deviations whereas we specify ours in level deviations. Our calibration is equivalent to their estimates for the response of the capital income tax rate at 0.39.

simulations are retained. This corresponds to an interval of [0.24 1.29] for $\frac{b_{-1}}{4y_0}$. The state distribution at $t = 0$ then consists of k_{-1}'s and b_{-1}'s in the retained simulations.

The empirical distribution of capital and debt in U.S. data has two features, largely captured by our simulated distribution. First, capital and debt are negatively correlated historically (see Figure 2).[8] Our simulation has a correlation coefficient of -0.5 over 100 periods across simulations, compared to -0.4 in the data from mid-1970s to 2014. Second, capital as a share of output fluctuates less than the corresponding debt share in the data. From 1947 to 2014, the capital-GDP ratio fluctuates between -10 and 14 percent of the sample mean, compared to between -14 and 9 percent in the simulated distribution; the debt-GDP ratio is between -47 and 117 percent of the sample mean, compared to -53 and 146 percent in the simulated distribution. Figure 3 presents the scatter plot of the state distribution for capital- and debt-output ratio at $t = 0$ ($s_0^b = \frac{b_{-1}}{4y_0}$ and $s_0^k = \frac{k_{-1}}{4y_0}$), where the vertical lines at $s_0^b = 0.40$ and $s_0^b = 0.85$ are the cutoffs for our defined low-debt and high-debt groups, roughly 35 and 10 percent of the retained simulations.[9] Figure 4 presents the state density plot. These two figures show that capital and debt in our state distribution are largely negatively correlated, but a substantial variation in the capital-output ratio exists for a given value of the debt-output ratio, driven by different shock histories.

3.2. The Baseline Analysis

This section analyzes government spending effects in high- and low-debt states using the baseline specification. To pick the initial states for assessing government spending

[8] Here we use federal debt held by the public divided by GDP as published in the supplement data of Congressional Budget Office (2015) for the data back to 1929.

[9] The cutoff point for the high-debt group at 0.85 might seem quite high. Based on the long-term budget outlook projected by Congressional Budget Office (2015), if accounting for the possible legislative changes, the federal debt-to-output ratio is projected to be persistently above 0.85 after 2024.

effects, we first divide the simulations into the low-debt group ($s_0^b < 0.40$) and the high-debt group ($s_0^b > 0.85$). Next, we compute the the average capital-output share within each debt group. In the high-debt group, this share is 10 percent below the distribution mean, and in the low-debt group, it is 4 percent above the distribution mean. For each group, we choose a simulation that has the capital-output ratio being closest to the corresponding average.

At time 0, the government increases its spending by one-percentage point of steady-state output, following the AR(1) process in equation (10). In Figure 5, the solid lines are the responses in a low-debt state and the dashed lines are those in a high-debt state. Unless specified in parentheses, the variables in Figure 5 are measured by the gaps between paths with and without the government spending shock relative to their steady-state levels in percent. Table 2 reports the cumulative government spending multipliers at different horizons conditional on the mean s_0^k in each debt group. The multipliers are computed by $\frac{\sum_{i=1}^{k} {r_{t+i-1}}^{-1} \triangle y_{t+i-1}}{\sum_{i=1}^{k} {r_{t+i-1}}^{-1} \triangle g_{t+i-1}}$, where $\triangle y$ and $\triangle g$ are level changes relative to the path without a government spending increase. When computing consumption or investment multipliers, $\triangle y$ is replaced by $\triangle c$ or $\triangle i$. The numbers in the brackets indicate the 95-percent intervals of cumulative multipliers computed for each state combination in the two debt groups.

Figure 5 shows that, regardless of debt states, an increase in government spending has a short-run expansionary effect on output, consumption, and labor, but a negative effect on investment and, hence, capital. The positive responses of consumption are driven by the complementarity between government spending and consumption. As households also derive utility from government spending, a fiscal expansion increases marginal utility of consumption, leading households to cut investment and boost consumption. Despite the qualitative similarities between the two sets of responses, the quantitative differences are nontrivial. Table 2 shows that the impact multiplier for output in a low-debt state is 47 percent higher than that in a high-debt state (0.85 vs. 0.58). The difference for the consumption multiplier is even bigger: the impact consumption multiplier in a low-debt state is more than twice as big as in a high-debt state (0.67 vs. 0.28). In addition, the cumulative five-year multiplier for consumption remains positive in a low-debt state but turns negative in a high-debt state.

Comparing the path with a government spending shock to that without one, the changes in the tax rate are the same at time 0, as the tax rate only responds to the lagged debt. The macroeconomic responses, nevertheless, diverge on impact, indicating some policy expectations effect. Expecting higher income taxes implies a decline in lifetime disposable income, which generates a negative wealth effect, discouraging current consumption but encouraging investment to smooth future consumption loss. This negative wealth effect is stronger in a high-debt state as households expect a bigger tax increase, making the initial increase in consumption and the initial decrease in investment smaller than in a low-debt state.

Intuitively, labor should increase more in a high-debt state because of a stronger negative wealth effect than in a low-debt state, but we observe the opposite here. Recall that the baseline specification with the hybrid GHH preference has a weak wealth effect on labor. The dominant channel here, instead, is the intertemporal substitution effect. Lower consumption demand in a high-debt state makes the real interest rate (computed as $\frac{1}{q_t}$) rise less than in a low-debt state to clear the goods market. A lower real interest rate decreases the opportunity cost of leisure in terms of goods units next period, $(1+r_t)w_t$. With a smaller increase in labor at time 0, output on impact is lower in a high-debt than in a low-debt state.

For the longer horizon, debt affects government spending effects mainly through fiscal adjustments. The expansionary effects of a government spending increase are counteracted by realized higher income tax rates. As the tax rates increase by a larger magnitude in a high-debt state than in a low-debt state, not surprisingly, higher income tax rates discourage investment and work. Moreover, the convexity of taxation costs implies that, for a given increase in the income tax rate, it has stronger effects in discouraging labor supply and investment when tax *levels* are higher. With less capital and labor, output in a high-debt state is lower in the longer horizon than in a low-debt state. Despite the distorting tax increases, the five-year cumulative output multiplier remains positive at 0.11 in a low-debt state, compared to -0.32 in a high-debt state.

Our baseline analysis shows that, on average, a high-debt level can make government spending less expansionary in both short and long runs. In addition, given different combinations of initial capital and debt in our simulations, we compute multipliers for each simulation path and obtain 95-percent intervals. Table 2 shows that the confidence band for the impact output multiplier is [0.75, 0.87] in the low-debt group and is [0.55, 0.67] in the high-debt group, and the 95-percent intervals for the five-year multiplier are [0.01, 0.13] and [-0.38, -0.19], respectively. The baseline analysis suggests a significant difference in fiscal multipliers depending on debt levels.

3.3. Wealth Effect on Labor Supply: the KPR Preference

While the baseline analysis supports the conventional view concerning debt-dependent government spending effects, this theoretical result is not robust. We now investigate several channels that can change the magnitudes of debt-dependent fiscal multipliers. To explore the role of wealth effects on labor, we use the KPR preference with $\psi = 1$ in equation (4). Figure 6 compares the responses to a government spending shock for high-debt and low-debt states under the KPR preference. To make the responses comparable to those in the baseline analysis, the high- and low-debt groups have the same initial debt and capital levels as those in the baseline analysis (Figure 5).

Comparing Figures 6 to 5 reveals two differences. First, government spending is less expansionary with the KPR, relative to the GHH, preference. Second, the impact labor response in a high-debt state becomes slightly larger than in a low-debt state. Both differences are related to the wealth channel on labor. As under the baseline analysis, private consumption increases due to its complementarity with government spending. With the KPR preference, a strong positive wealth effect induced by higher consumption lowers incentives to work more in both debt states, making government spending less expansionary than the baseline analysis. On the other hand, as households expect a larger tax increase in a high-debt state with the KPR preference, the negative wealth effect induces them to work more in a high-debt state than in a low-debt state to smooth future consumption. The short-run difference in the output multiplier, therefore, diminishes between two debt states, as shown in the upper panel of Table 3.[10] While the output multipliers are similar in the short run, government spending has a more negative impact on output in a high-debt state than in a low-debt state at the longer horizon.

Empirically, the importance of the wealth effect on labor supply is inconclusive. House and Shapiro (2006) attribute the delayed recovery in employment to the positive wealth effect from expecting future cuts in income tax rates introduced by the 2001 and 2003 tax

[10] In a nonlinear model, the change in output depends both on the change in labor and the predetermined capital level. For the same change in labor, a lower predetermined capital level reduces the output change. That is why even though the initial labor change is slightly more positive, the initial output change is less positive in a high-debt state than in a low-debt state, as the predetermined capital effect dominates.

laws, but many point to the structural changes in the U.S. labor market (e.g., Groshen and Potter (2003) and Autor (2010)). In a thorough survey, Keane (2011) notes that the large literature on the tax changes and labor supply is best characterized by considerable controversy. Thus, the empirical importance that debt can affect government spending effects through the wealth effect on labor remains an open question.

3.4. Fiscal Adjustment Scheme: Government Spending Reversal

The baseline analysis assumes that the government adjusts only the income tax rate to stabilize debt. In reality, fiscal adjustments are often accomplished by multiple instruments. Devries et al. (2011) document discretionary fiscal consolidations for 17 OECD countries from 1978 to 2009. For the U.S., nine of 15 episodes rely on both tax increases and spending cuts, five on tax increases only, and one on spending cuts only. We now revise the fiscal rules in (10) and (11) to allow for both spending and the income tax adjustments:

$$g_t = (1 - \rho_g)\, g + \rho_g g_{t-1} - \gamma_g \phi\, (b_{t-1} - b) + \varepsilon_t^g; \qquad \tau_t = \tau + \gamma_\tau (1 - \phi)(b_{t-1} - b), \quad (12)$$

where ϕ measures the share of adjustments by spending cuts. We assume $\phi = 0.5$: in each period roughly half of the debt reduction is through tax cuts and half is through spending cuts.[11]

When spending and taxes are used for fiscal adjustments, the short-run difference in output and consumption becomes smaller between two debt states. In a high-debt state, expecting more government spending cuts induces a stronger positive wealth effect, offsetting the negative wealth effect from expecting larger tax increases. As under the KPR preference, the short-run output multipliers in both debt states become closer, while

[11] This implies $\gamma_\tau (1 - \phi) = 0.014$ and $\gamma_g \phi = 0.002$.

the differences in the longer horizon remain large. The bottom panel of Table 3 shows that the cumulative five-year output multiplier is -0.25 in a high-debt state, compared to about 0 in a low-debt state.

The results under two alternative specifications, one with the KPR preference and the other with partial spending reversals, show that debt-dependent government spending effects are better supported by theory over a longer horizon. The difference in the short-run multipliers can be small, which offers one potential explanation for inconclusive empirical evidence on debt-dependent government spending effects.

4. The Role of the Capital State

The analysis so far sets aside the issue of the capital state by conditioning the analysis on the mean capital-output ratio in each debt group, as shown in Figure 5. The 95-percent intervals in Table 2 suggest that the differences in multipliers across debt groups, however, may change once accounting for capital differentials. In this section, we examine the importance of the capital state in government spending effects, and show that capital variations add to the difficulty in establishing empirical evidence for debt-dependent spending effects.

4.1. Capital-Dependent Government Spending Effects

To evaluate the impact of capital on government spending effects, we assess government spending effects conditional on different capital-output ratios. Using the baseline specification, Table 4 summarizes the cumulative multipliers under the maximum and minimum capital state value (s_0^k) in the high-debt group. The maximum (minimum) s_0^k corresponds to -0.7 (-19.4) from the entire distribution mean.[12] In general, a higher

[12] Since capital and debt states are negatively correlated on average, almost all capital states simulated at time 0 are below the distribution mean in the high-debt group.

initial capital-output ratio makes government spending more expansionary than does a lower capital-output ratio. This is because more capital implies a higher marginal product of labor and, hence, a smaller cost to raise consumption in response to a government spending increase. In addition, it lowers the marginal product of capital, making it less costly to cut investment and to accommodate higher consumption in the short run. Thus, between the two capital-output ratios, the output multiplier is bigger with a higher capital-output state. The impact output multiplier is 0.64 with the maximum s_0^k and 0.53 with the minimum s_0^k, while the five-year cumulative output multiplier is -0.21 with the maximum s_0^k and -0.42 with the minimum s_0^k. It implies that the initial capital-output ratio has a larger impact in the long run than short run.

4.2. Empirical Implications

Theoretical analysis on the importance of the capital state suggests that conditioning on a single state may be insufficient to identify debt-dependent policy effects. Estimates of debt-dependent government spending effects often involve sorting samples by some measure of government indebtedness, such as the average debt-output ratio over the sample period (e.g., Ilzetzki et al. (2013)), or include some debt-related dummy variable to indicate good or bad fiscal regimes (e.g., Perotti (1999)). Using our simulated data series between $t = -99$ and $t = 0$ from the baseline specification, we estimate government spending effects using structural VARs for each simulation contained in the state distribution.

We adopt two common identification strategies for estimating fiscal policy effects with structural VARs. First, we use data on government spending, output, and consumption ($[G, Y, C]$), and assume that government spending does not contemporaneously respond to output and consumption by applying a Cholesky decomposition to the contemporaneous matrix, similar to Fatás and Mihov (2001) and Ilzetzki et al. (2013).[13] This identifying assumption agrees with our data generating process that government spending is exogenous in the baseline specification. Second, we use data on government spending, tax revenues, and output ($[G, T, Y]$), and also government spending does not contemporaneously respond to other variables. Following Blanchard and Perotti (2002), we impose a unit tax revenue elasticity with respect to output on the contemporaneous matrix, since the income tax rate in our model is flat.

The left column of Figure 8 presents estimation results for the system $[G, Y, C]$ and the right column is for $[G, T, Y]$. Following the theoretical analysis, the cutoff value for the low-debt (high-debt) state is for a debt-output ratio to be less than 0.40 (above 0.85). The top row has the results dividing the high-debt and low-debt groups based on the initial debt (as in Kirchner et al. (2010)); the middle row has the results based on the ending value; and the bottom row has the results based on the average sample value (as in Ilzetzki et al. (2013)). The units are transformed to periodic (non-accumulative) multipliers, as commonly done in the empirical literature. The solid lines are mean responses and dotted-dashed lines are two standard-deviation (95-percent) intervals, based on the estimates of 4749 simulations, with outliers removed.

Of all the estimations, only the mean responses conditional on the average debt-output ratio under the $[G, Y, C]$ (the left bottom plot of Figure 8) have output respond less positively on impact in the high-debt group than in the low-debt group. The confidence bands, however, largely overlap between the two debt groups. Figure 8 makes clear that, regardless of which sorting criterion is adopted, one cannot establish clear evidence on

[13] Since we do not have open-economy variables, our system only includes output and the two GDP components: government spending and consumption.

debt-dependent government spending effects, even though the model used to generate the data (the baseline specification) produces significantly different output multipliers between the two debt groups. Note that the 95-percent confidence intervals of the two debt groups for the multipliers in Table 2 do not overlap.

Our estimation results echo Bi et al.'s (2013) conclusions that the effects of fiscal consolidations depend on various factors—including debt levels, monetary policy, and the composition of fiscal consolidation instruments. With simulated data, our paper extends their conclusion: even if debt-dependent government spending effects exist in data, the interference between endogenous states can make empirical efforts to recover state-dependent effects difficult.

5. Uncertain Timing and Adjustment Magnitudes

Our analysis is so far conducted assuming that there is no uncertainty surrounding fiscal adjustments. Under such rules, households know when and by how much fiscal instruments respond to debt. While these types of rules are common in the literature (see, e.g., Forni et al. (2009), Uhlig (2010), Zubairy (2014), and Traum and Yang (2015)), fiscal adjustments are subject to uncertainty regarding timing and the composition of adjustment instruments (Bi et al. (2013)), the debt target that a government aims to stabilize in the long run (Richter and Throckmorton (2015)), and so on. To explore uncertain timing and adjustment magnitudes, we revise the income tax rule in (11) as follows:

$$\tau_t = \tau + \gamma_{\tau,t} \left(b_{t-1} - b \right), \tag{13}$$

where $\gamma_{\tau,t}$ can take one of two values:

$$\gamma_{\tau,t} = \begin{cases} \gamma, & \text{if } s_t^b < s_t^{b*}, \\ \gamma^H, & \text{if } s_t^b \geq s_t^{b*}, \end{cases}$$

where $\gamma^H > \gamma > 0$. The higher adjustment magnitude, γ^H, represents an aggressive adjustment (or a consolidation), which is triggered if the debt level exceeds some threshold value, s_t^{b*}. This debt threshold is stochastic and drawn from an exogenous distribution, $s_t^{b*} \sim \mathcal{B}^*$. The stochastic debt thresholds is to reflect political uncertainty in conducting fiscal consolidations. We follow Davig et al. (2010) and model the cumulative density function of the debt threshold distribution as a logistic function with parameters η_1 and η_2,[14]

$$p_t \equiv P\left(s_t^b \geq s_t^{b*}\right) = \frac{\exp(\eta_1 + \eta_2 s_t^b)}{1 + \exp(\eta_1 + \eta_2 s_t^b)}, \tag{14}$$

where p_t is the regime-switching probability associated with the debt level of $\frac{b_{t-1}}{4y_t}$. The higher the debt, the more likely the government would undertake fiscal consolidations. Figure 9 presents the cumulative density function of the debt threshold distribution. For example, when the debt-output ratio is 0.85, the probability of conducting a fiscal consolidation is about 0.3, and when the ratio is 1.10, the probability is almost 1.

Starting from $t = 1$, the government determines whether to adopt γ^H by drawing debt threshold from $\mathcal{B}^*$. If $s_1^b > s_1^{b*}$, $\gamma_{\tau,1}$ switches from 0.027 to $\gamma^H = 0.054$; otherwise, the existing process continues and a new threshold is drawn at $t = 2$ to again be compared to s_2^b. The process continues until a consolidation is adopted. Once adopted, the probability of staying at the fiscal consolidation regime is $prob\left(\gamma_{\tau,t} = \gamma^H | \gamma_{\tau,t-1} = \gamma^H\right) \equiv p^H$, while the probability of switching back to the no-consolidation regime is $1 - p^H$. We set

[14] Davig et al. (2010) specify the distribution in terms of a tax rate, while we specify it in terms of the debt level.

$p^H = 0.9375$, implying that, on average, a consolidation continues for about four years, roughly the average length of consolidation episodes documented in Devries et al. (2011) for 17 OECD countries from 1978 to 2009.

Figure 10 plots the cumulative multipliers under scenarios with and without policy uncertainty in the high- and low-debt groups. The solid lines are the responses under the baseline specification, with no policy uncertainty; the dashed lines are the responses with uncertain consolidation timing and adjustment magnitudes in the income tax rate, as described by (13). The thick lines are the mean responses and the thin lines are the 5-th and 95-th confidence bands for each group.

The theoretical confidence bands in Figure 10 are generated from several sources. First, simulations from different points in the state distribution of capital and debt at time 0 lead to different output responses. Second, under policy uncertainty, additional response variations in the high-debt group come from uncertain fiscal adjustment paths due to the regime-switching process for the income tax rate. In the low-debt group, since the probability of a fiscal consolidation is almost zero, policy uncertainty makes little difference (see Figure 9).

When debt levels are high, policy uncertainty is, on average, contractionary, which can be seen by comparing the thick lines in the right plot. Introducing uncertain timing and magnitudes of fiscal adjustments induces expectations of higher future tax rates on both capital and labor income. As shown in Yang (2005), these two expectations effects have opposite signs on current labor, investment, and output. Expecting higher capital income tax rates discourages current investment, which lowers the marginal product of labor and reduces output. In contrast, expecting higher labor income tax rates reduces current consumption and so increases investment due to the negative wealth effect. The opposite signs of responses generate an overall small effect from the policy expectation channel as the general income tax adjustment affects both capital and labor income tax rates. The regime-switching tax rule, however, generates a variety of possible fiscal adjustment paths, which substantially widen the confidence bands under policy uncertainty in the high-debt group. Such wide bands increase difficulty to estimate the debt-dependent government spending effects, as output response differences between two debt states can be smaller for those responses near the upper bound in the high-debt group.

6. Conclusion

We study debt-dependent effects of fiscal expansions in a neoclassical growth model that is solved nonlinearly under rational expectations. Our analysis, assuming the hybrid GHH preference and income tax adjustments, supports the conventional view that government spending is less expansionary when it is highly indebted. In a high-debt state, expecting higher future taxes implies a stronger negative wealth effect on consumption, weakening the short-run stimulative effect of a government spending increase. For a longer horizon, both a higher tax level and a larger increase in tax rates make investment and labor respond more negatively in a high-debt state than in a low-debt state, producing a more negative output response.

The result that a high-debt state makes government spending less expansionary, however, is not robust to alternative specifications. When the wealth effect on labor supply is sufficiently strong, such as with the KPR preference, or when fiscal adjustments involve government spending reversals, the short-run difference in debt-dependent government spending effects can be quite small. In particular, as the share of fiscal adjustments via government spending cuts gets large, it is possible that government spending becomes more expansionary in a high-debt state than in a low-debt state in the short run. Expecting government spending reversals produces a positive wealth effect, which can dominate the negative wealth effect from expecting higher future taxes and thus reverse the original negative consumption response in a high-debt state.

Our analysis of a high-debt state requires simulations of government spending effects from a point off the steady state, differing from most analyses which start at a deterministic steady state. The analysis raises an issue of the relationship among state variables, which are jointly affected by historic shocks. When accounting for the endogenous relationships between debt and capital, we find that the initial capital level at the time of a shock plays an important role in government spending effects. Common empirical approaches to sorting samples based on a single debt state are likely to fail to recover debt-dependent government spending effects. Both the possibility of a small difference in government spending effects in data and the failure to account for other states contribute to the mixed empirical findings on debt-dependent government spending effects.

One caveat is that our analysis does not consider rising sovereign risk premia when the debt level approaches a country's fiscal limits, beyond which a sovereign default is likely to occur. Policy expectations for countries with a sovereign default history can go beyond fiscal adjustments or consolidations considered here; they may involve expectations about a future period of severe economic disruption, including turmoil in financial systems, trade exclusions, etc. Under those circumstances, expectations effects are likely to be much more contractionary on government spending effects than presented here.

Acknowledgment We thank Giancarlo Corsetti, Shen Guo, Juan Carlos Hatchondo, Ethan Ilzetzki, Robert Kollmann, Eric Leeper, Campbell Leith, Bruce Preston, Werner

Roeger, Christopher Sims, Nicolas Magud, David Savitski, Andreas Tudyka, Harald Uhlig, and participants at the 2015 Tsinghua-CAEPR Conference on Monetary and Fiscal Policy, the Conference on the Post-Crisis Slump in the European Commission, and various seminars for helpful comments. Yang acknowledges the support from the Ministry of Science and Technology of Taiwan. The views expressed here are those of the authors and do not necessarily represent those of Federal Reserve Bank of Kansas City, the Federal Reserve System, the IMF, or IMF policy.

	parameters or steady-state variables	
β	the discount factor	0.99
θ	corresponding to a labor supply elasticity of 0.5	3
ψ	corresponding to the GHH preference	0
δ	capital depreciation rate for capital	0.025
α	capital income share	0.36
κ	investment adjustment cost parameter	1.7
a	normalized TFP	1
ν	elasticity of substitution b/w c_t and g_t	0.42
τ	income tax rate	0.31
$\frac{b}{4y}$	debt-output ratio	0.373
$\frac{g}{y}$	government spending-output ratio	0.208
$\frac{z}{y}$	government transfers-output ratio	0.086
γ_τ	income tax response to debt	0.027
ρ_g	AR(1) coefficient for g_t	0.87
ρ_a	AR(1) coefficient for a_t	0.79
ρ_z	AR(1) coefficient for z_t in the stationary regime	0.96
μ	coefficient for z_t in the stationary regime	1.006
p_1^z	regime-switching parameter for transfers	0.992
p_2^z	regime-switching parameter for transfers	0.992
σ_g	standard deviation of ε^g	0.014
σ_a	standard deviation of ε^a	0.007
σ_z	standard deviation of ε^z	0.0435

Table 1: **Baseline calibration and some steady-state values**.

	low-debt state	***high-debt state***
	output multiplier	
impact	0.85	0.58
	[0.75, 0.87]	[0.55, 0.67]
1 year	0.74	0.46
	[0.65, 0.76]	[0.43, 0.55]
5 years	0.11	−0.32
	[0.01, 0.13]	[-0.38, -0.19]
	consumption multiplier	
impact	0.67	0.28
	[0.55, 0.70]	[0.23, 0.42]
1 year	0.60	0.20
	[0.49, 0.63]	[0.15, 0.34]
5 years	0.17	−0.34
	[0.06, 0.20]	[-0.40, -0.17]
	investment multiplier	
impact	−0.82	−0.70
	[-0.83, -0.80]	[-0.74, -0.68]
1 year	−0.86	−0.74
	[-0.87, -0.84]	[-0.69, -0.72]
5 years	−1.06	−0.99
	[-1.06, -1.05]	[-1.02, -0.97]

Table 2: **Cumulative multipliers: the baseline specification**. The numbers in brackets represent the 95-percent intervals.

	low-debt state	***high-debt state***
	the KPR preference	
impact	0.28	0.26
1 year	0.16	−0.13
5 years	−0.52	−0.66
adjustments by tax increases and spending reversals		
impact	0.79	0.64
1 year	0.70	0.56
5 years	0.02	−0.25

Table 3: **Cumulative multipliers: alternative specifications.**

	maximum s_0^k	*minimum* s_0^k
	output multiplier	
impact	0.64	0.53
1 year	0.53	0.41
5 years	−0.21	−0.42
	consumption multiplier	
impact	0.43	0.20
1 year	0.32	0.11
5 years	−0.19	−0.45
	investment multiplier	
impact	−0.75	−0.66
1 year	−0.79	−0.70
5 years	−1.02	−0.97

Table 4: **Cumulative multipliers: different capital states in the high-debt group.** The maximum s_0^k in the high-debt group is −0.7 percent and the minimum is −19.4 percent from the distribution mean.

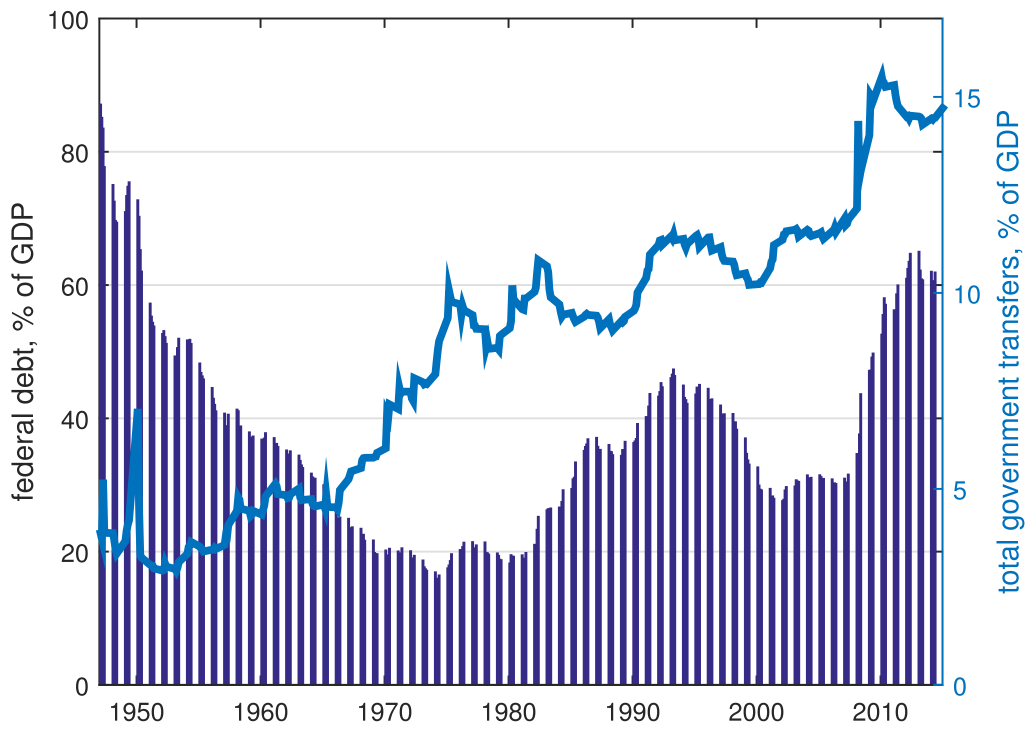

Figure 1: **U.S. fiscal data.** Debt data are computed from the end-period values for the market value of privately held gross federal debt (published by Federal Reserve Bank of Dallas). Transfers data are computed from social benefits of total government (NIPA Table 3.1, line 23).

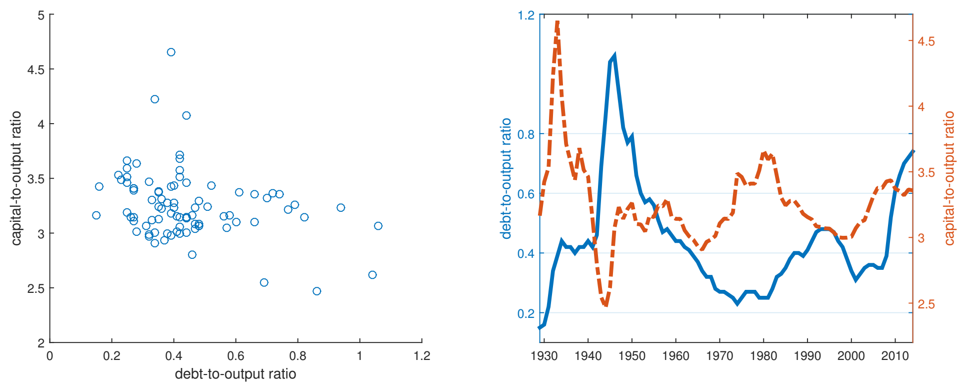

Figure 2: **Empirical distribution of capital and debt.** See footnotes 6 and 8 for data sources.

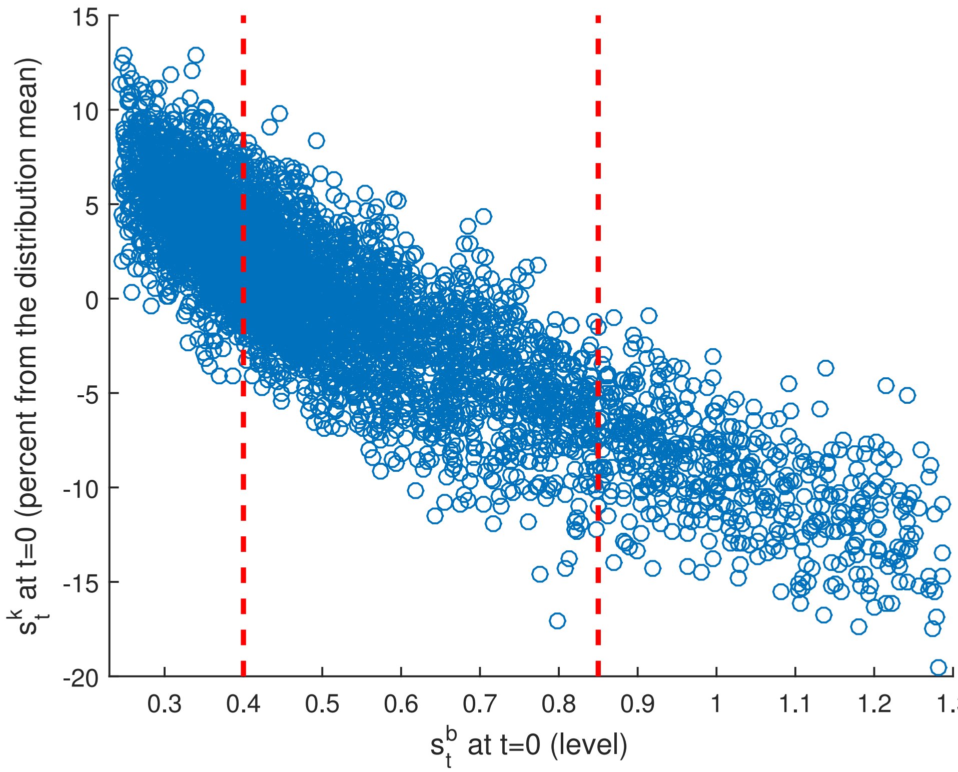

Figure 3: **The simulated state distribution at** $t = 0$**.** A debt-output ratio of 0.40 (0.85) is the upper bound (lower bound) of the low-debt (high-debt) group.

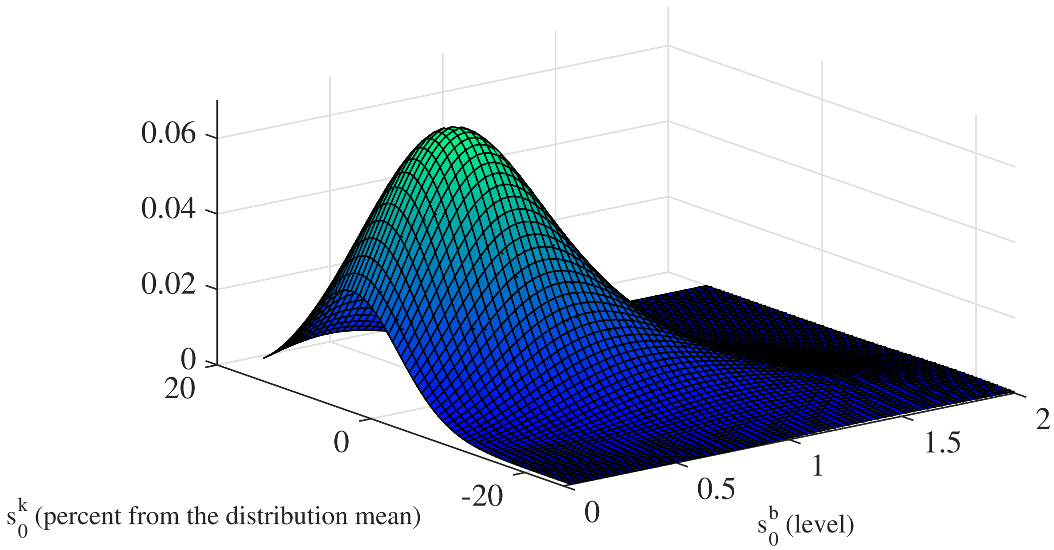

Figure 4: **The density of the state distribution at $t = 0$.**

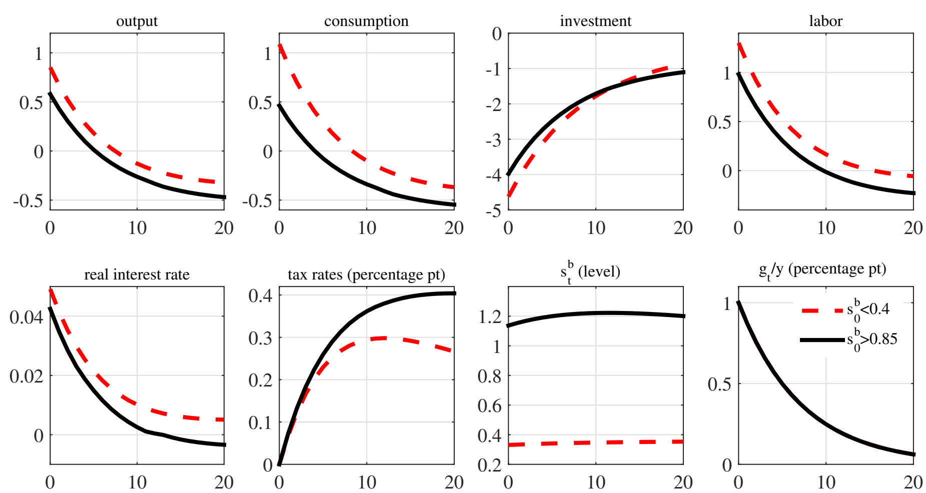

Figure 5: **Debt-dependent government spending effects: the baseline specification.** The x-axis is in quarters. Unless specified in parentheses, the y-axis measures gaps between paths with and without the shock relative to the steady state in percent.

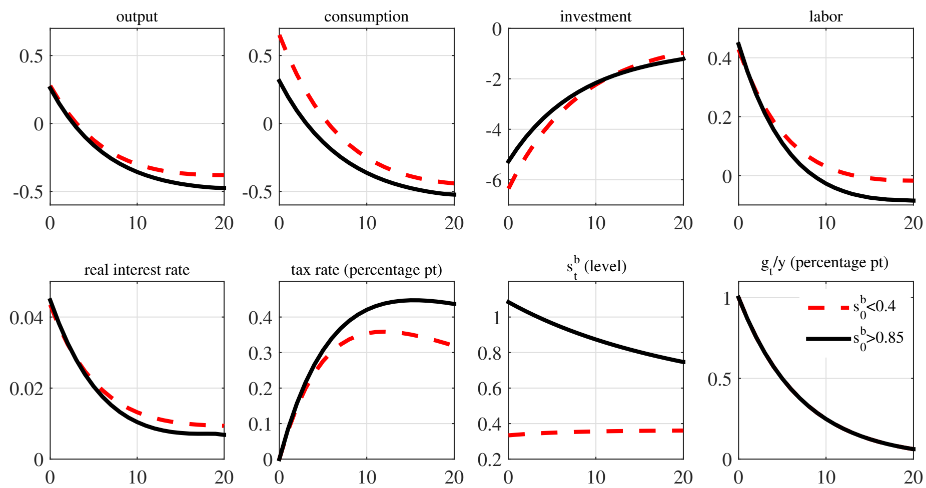

Figure 6: **Government spending effects: the KPR preference.** The x-axis is in quarters. Unless specified in parentheses, the y-axis measures gaps between paths with and without the shock relative to the steady state in percent.

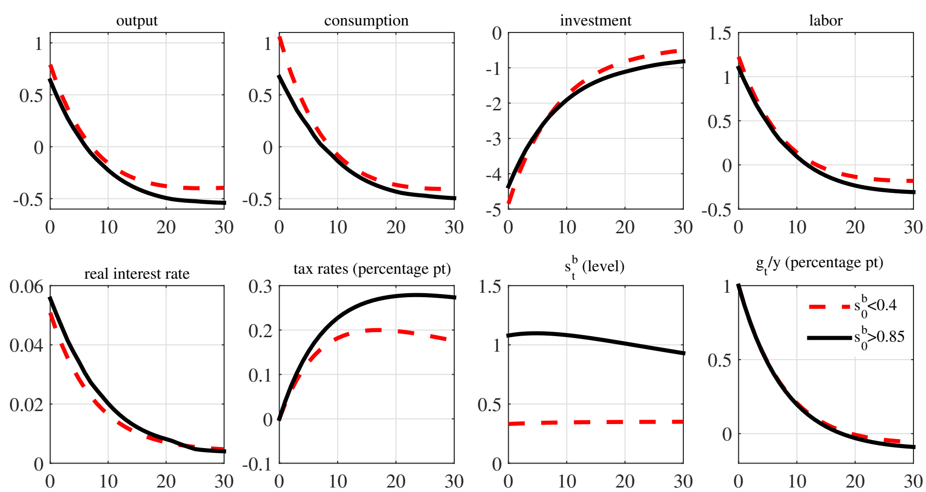

Figure 7: **Government spending effects when both taxes and government spending adjust.** The x-axis is in quarters. Unless specified in parentheses, the y-axis measures gaps between paths with and without the shock relative to the steady state in percent.

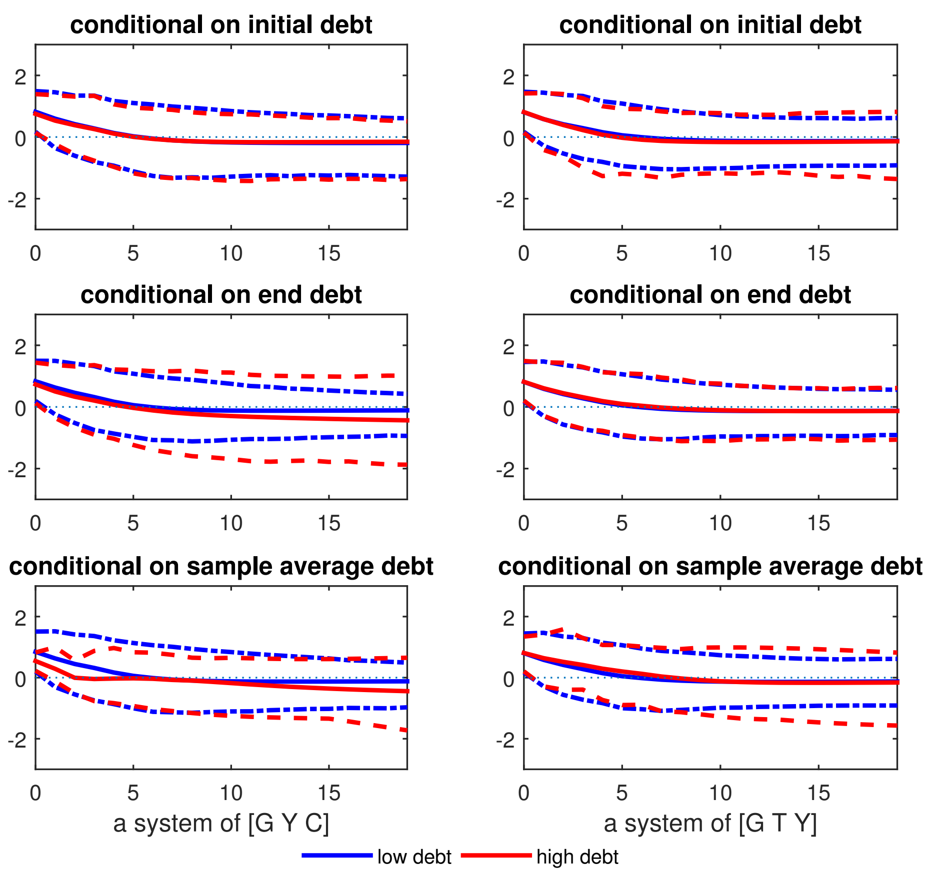

Figure 8: **Structural VAR estimation of government spending effects on output.** The left column imposes Cholesky decomposition; the right column imposes the correct tax revenue elasticity with respect to output. The solid lines are the mean responses. The error bands are the two standard-deviation bands.

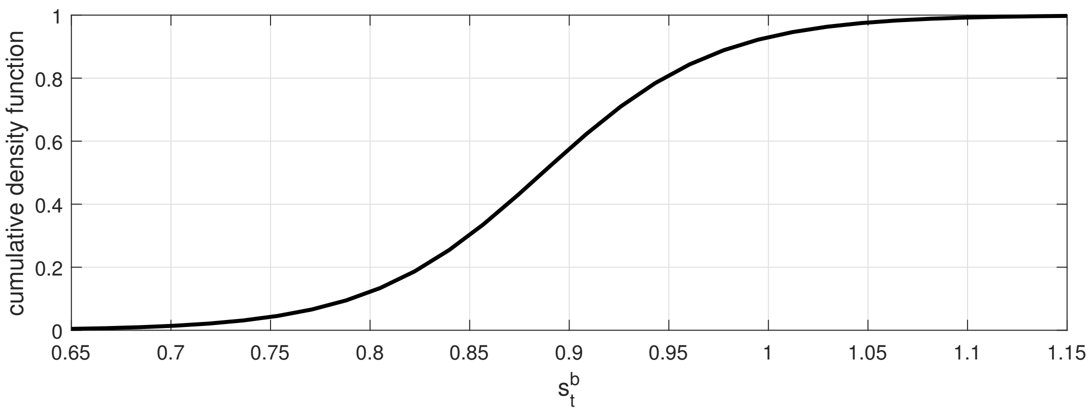

Figure 9: **The distribution of government debt thresholds.**

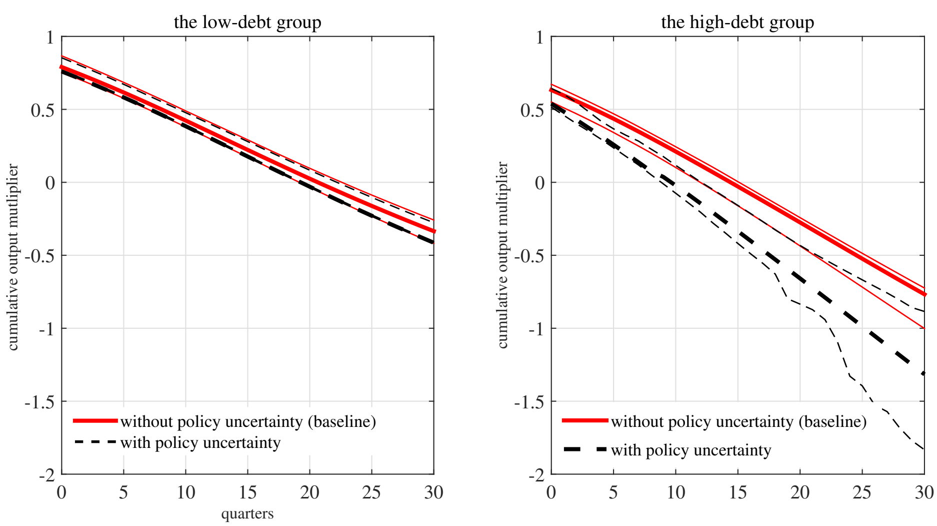

Figure 10: **Cumulative output multipliers of government spending, with and without uncertainty.** The scenario "without policy uncertainty" is the baseline specification. The scenario "with policy uncertainty" assumes uncertain timing and adjustment magnitudes of the income tax rate to debt. The thick lines are the mean responses and the light lines are the 5-95 percent bands in each debt group.

Appendix A. Equilibrium Conditions of the Baseline Specification

$$\tilde{c}_t = \left[\omega\left(c_t\right)^{\frac{\nu-1}{\nu}} + (1-\omega)\left(g_t\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{A.1}$$

$$\lambda_t = (\tilde{c}_t - \varphi l_t^{\theta})^{-\sigma}\omega c_t^{-\frac{1}{v}}\tilde{c}_t^{\frac{1}{v}} \tag{A.2}$$

$$(\tilde{c}_t - \varphi l_t^{\theta})^{-\sigma}\varphi\theta l_t^{\theta-1} = \lambda_t(1-\tau_t)w_t \tag{A.3}$$

The above two equations can be reduced as:

$$\varphi\theta l_t^{\theta-1} = \omega c_t^{-\frac{1}{v}}\tilde{c}_t^{\frac{1}{v}}(1-\tau_t)w_t. \tag{A.4}$$

Let λ_t and ξ_t be the Lagrangian multipliers for the household's budget constraint and the law of motion for capital. Also, define Tobin's Q as $TQ_t = \frac{\xi_t}{\lambda_t}$. Then,

$$1 = TQ_t\left(1 - \frac{\kappa}{2}\left(\frac{i_t}{k_{t-1}} - \delta\right)\right). \tag{A.5}$$

$$TQ_t = \beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau_{t+1})r_{t+1}^k + TQ_{t+1}\left((1-\delta) + \kappa\left(\frac{i_{t+1}}{k_t} - \delta\right)\frac{i_{t+1}}{k_t} - \frac{\kappa}{2}\left(\frac{i_{t+1}}{k_t} - \delta\right)\right)\right] \tag{A.6}$$

$$k_t = (1-\delta)k_{t-1} + i_t - \frac{\kappa}{2}\left(\frac{i_t}{k_{t-1}} - \delta\right)^2 k_{t-1} \tag{A.7}$$

$$\lambda_t q_t = \beta E_t\lambda_{t+1} \tag{A.8}$$

$$y_t = a_t k_{t-1}^{\alpha} l_t^{1-\alpha} \tag{A.9}$$

$$(1-\alpha)y_t = w_t l_t \tag{A.10}$$

$$\alpha y_t = r_t^k k_{t-1} \tag{A.11}$$

$$y_t = c_t + i_t + g_t \tag{A.12}$$

$$\tau_t\left(w_t l_t + r_t^k k_{t-1}\right) + q_t b_t = g_t + b_{t-1} + z_t \tag{A.13}$$

$$\tau_t = \tau + \gamma(b_{t-1} - b) \tag{A.14}$$

$$a_t = (1-\rho_a)a + \rho_a a_{t-1} + \varepsilon_t^a \tag{A.15}$$

$$g_t = (1-\rho_g)g + \rho_g g_{t-1} + \varepsilon_t^g \tag{A.16}$$

Appendix B. Solving the Model Nonlinearly

When solving the nonlinear model, the state space is $\mathbf{S}_t = \{b_{t-1}, a_t, g_t, z_t, k_{t-1}, rs_t^z\}$. Write the decision rules for the end-of-period debt as $b_t = f^b(\mathbf{S}_t)$ and consumption as $c_t = f^c(\mathbf{S}_t)$. The decision rules are solved as follows.

1. Define the grid points by discretizing the state space. Make initial guesses for f_0^b and f_0^c over the state space.
2. At each grid point, solve the nonlinear model and obtain the updated rules, f_i^b and f_i^c, using the given rules f_{i-1}^b and f_{i-1}^c:
   (a) Given b_{t-1}, derive τ_t using (A.14).
   (b) Compute $\tilde{c}_t$ from (A.1). Using equations (A.4) and (A.10), we can solve for l_t by the following equation

$$l_t = \left[\frac{\omega(\tilde{c}_t/c_t)^{1/v}(1-\tau_t)(1-\alpha)a_t k_{t-1}^{\alpha}}{\varphi\theta} \right]^{\frac{1}{\theta+\alpha-1}}.$$

   With l_t, λ_t can be computed from (A.2).
   (c) Compute y_t, w_t, and r_t^k using (A.9), (A.10), and (A.11).
   (d) Given y_t, c_t, and g_t, we can solve for i_t from the aggregate resource constraint (A.12). Then, obtain TQ_t, k_t, and q_t from (A.5), (A.7), and (A.13).
   (e) Use linear interpolation to obtain $f_{i-1}^b(\mathbf{S}_{t+1})$ and $f_{i-1}^c(\mathbf{S}_{t+1})$, in which $\mathbf{S}_{t+1} = (b_t, a_{t+1}, g_{t+1}, z_{t+1}, k_t, rs_{t+1}^z)$. If $rs_{t+1}^z = 1$, $z_{t+1} = (1-\rho_z)z + \rho_z z_t + \varepsilon_{t+1}^z$. If $rs_{t+1}^z = 2$, $z_{t+1} = \mu z_t + \varepsilon_{t+1}^z$. Then, follow the above steps to solve τ_{t+1}, r_{t+1}^k, λ_{t+1}, i_{t+1}, and TQ_{t+1}.
   (f) Update the decision rules f_i^b and f_i^l using (A.6) and (A.8).
3. Check convergence of the decision rules. If $|f_i^b - f_{i-1}^b|$ or $|f_i^c - f_{i-1}^c|$ is above the desired tolerance (set to $1e-7$), go back to step 2; otherwise, f_i^b and f_i^c are the decision rules.

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